Finding Tree Heights Using Trigonometry

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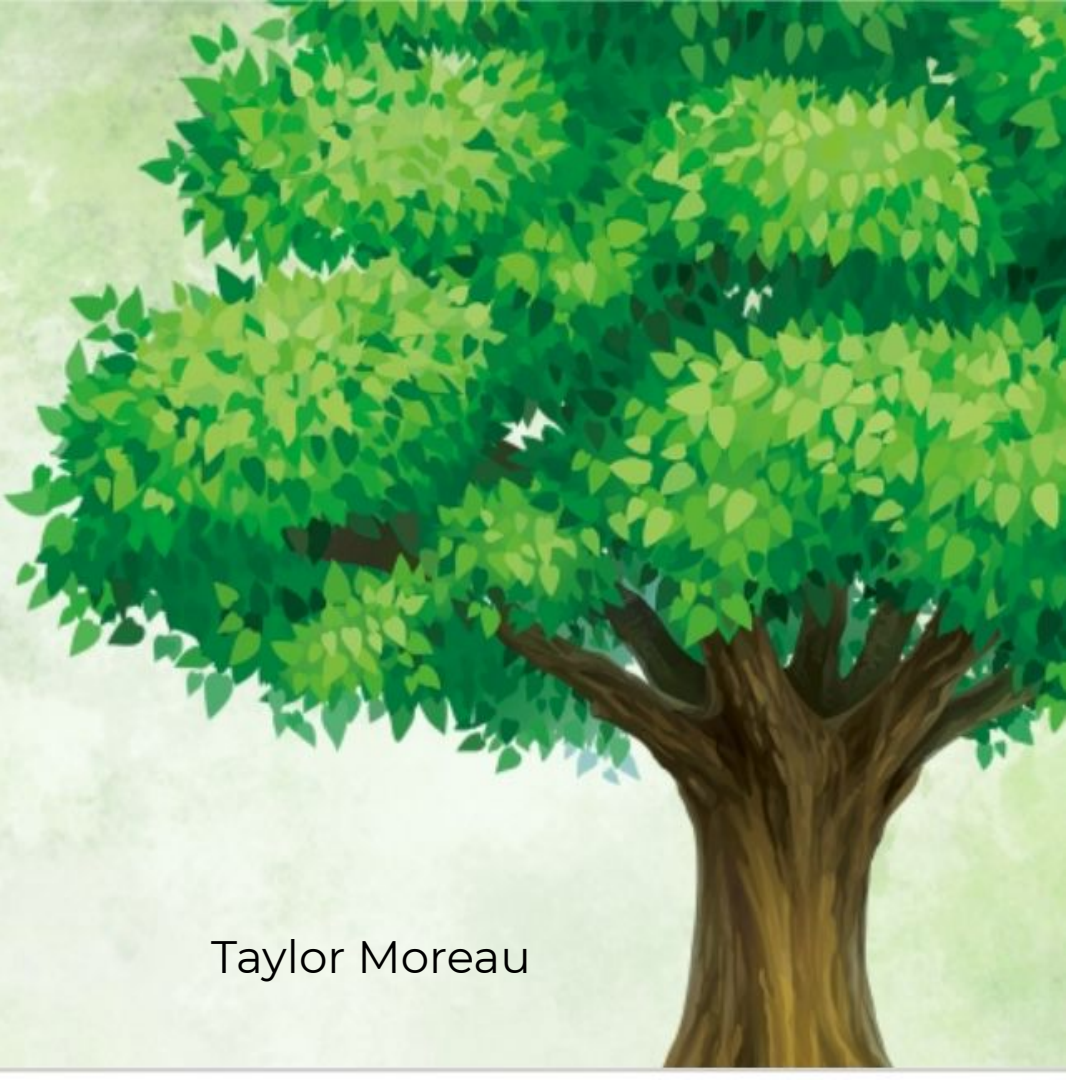
Introduction to the Clinometer Method and the Stick Method to finding the height of a tree



Taylor Moreau

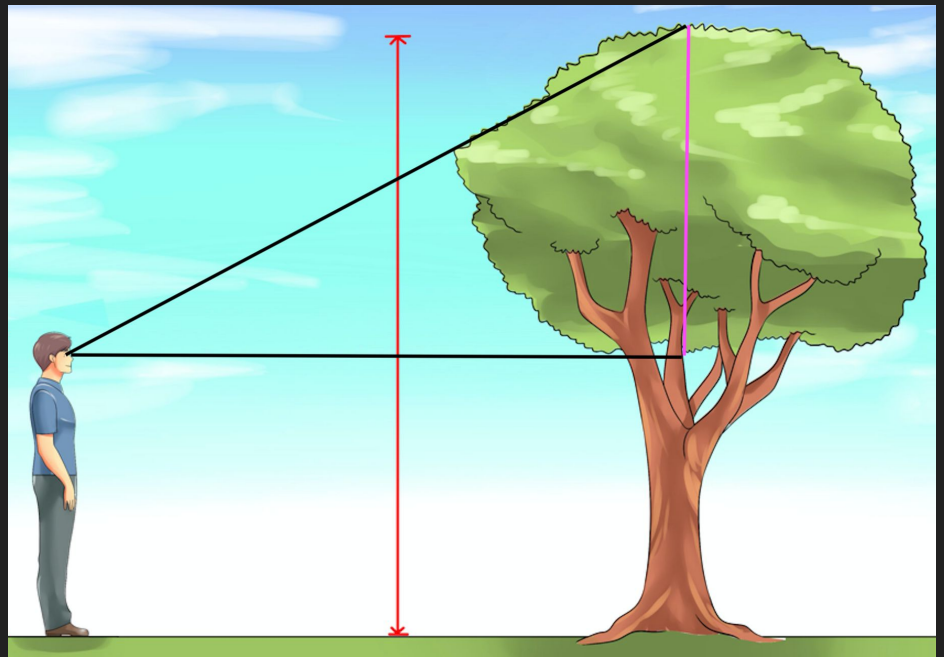
Lesson Objectives

- Solving to find heights of trees using Right Triangle Trigonometry and Similar Triangles
- Learn and use the **Clinometer Method** and the **Stick Method** to measure the height of a tree
- Hands-On Activity finding tree height with your own eye height
- Using trigonometric ratios in real-world applications

  (SOH-CAH-TOA)

Intro: Using Trigonometric Ratios

You are standing 42 ft away from the base of a tree. Your line of sight is 5 ft from the ground. If you measure the angle of elevation to the top of the tree to be 43°, how tall is the tree?



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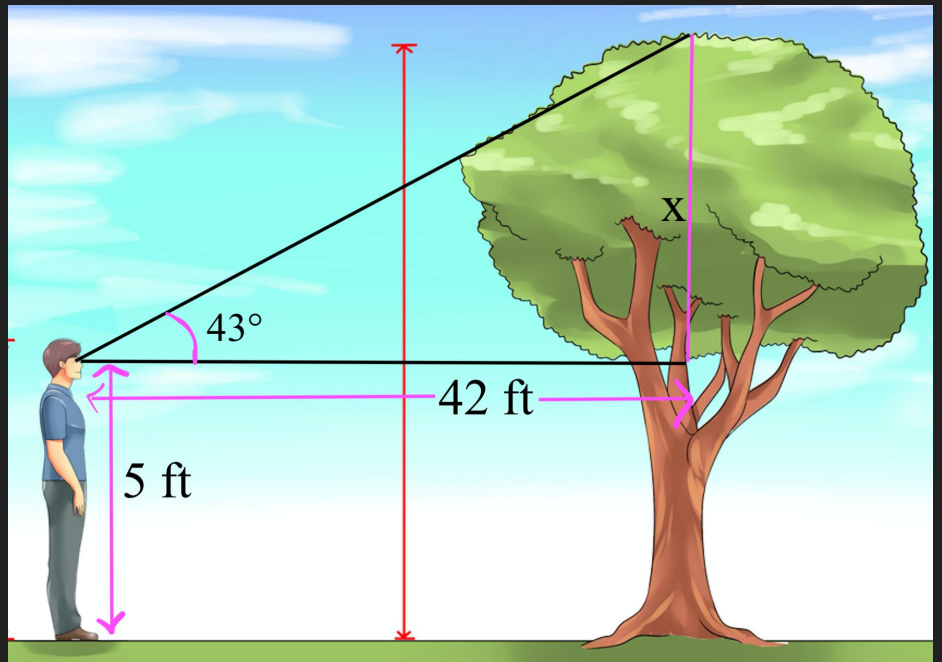

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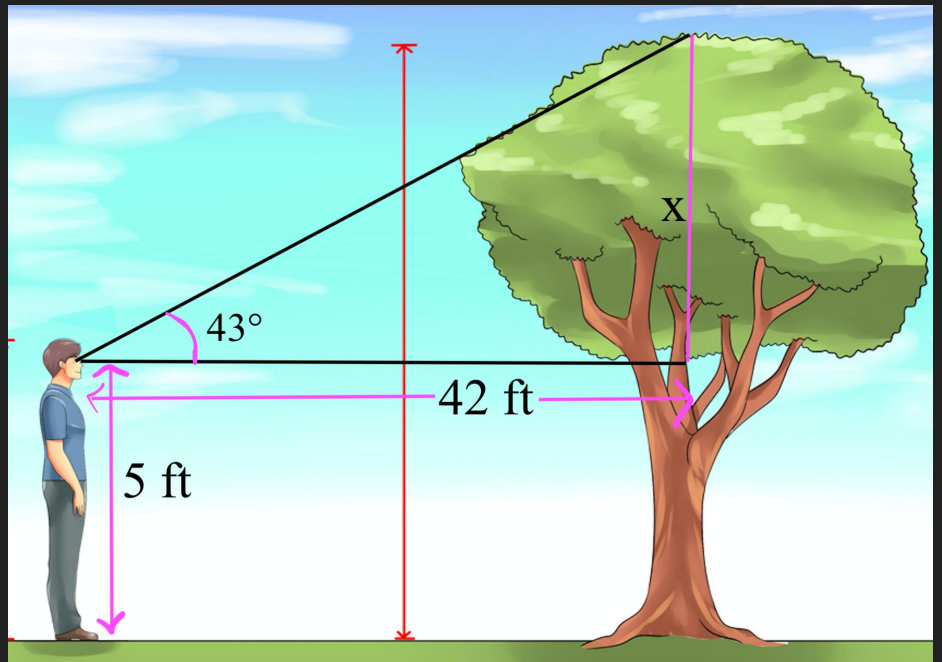

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$$C = \frac{A}{H}$$

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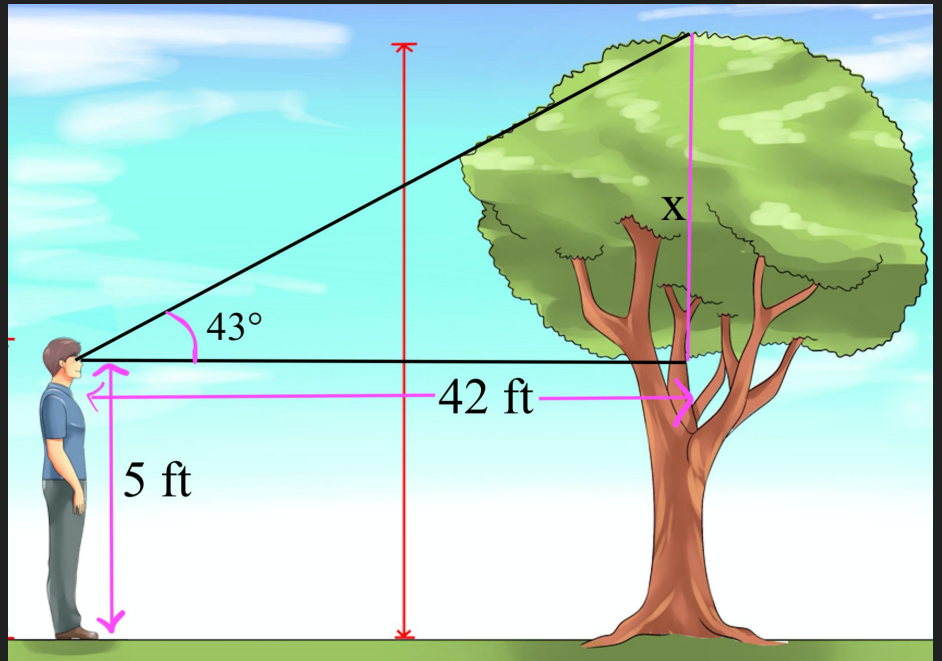

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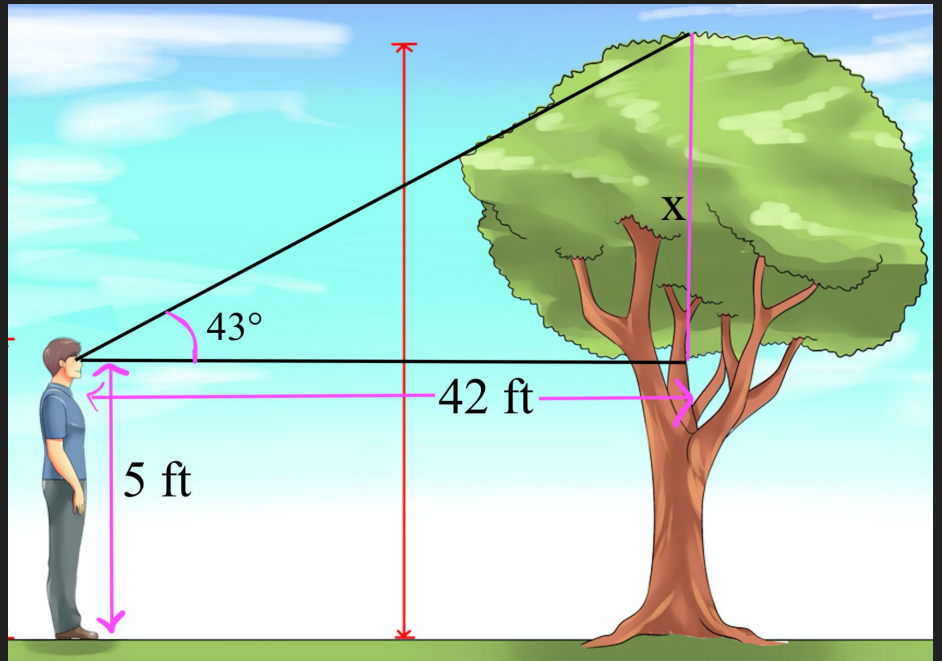

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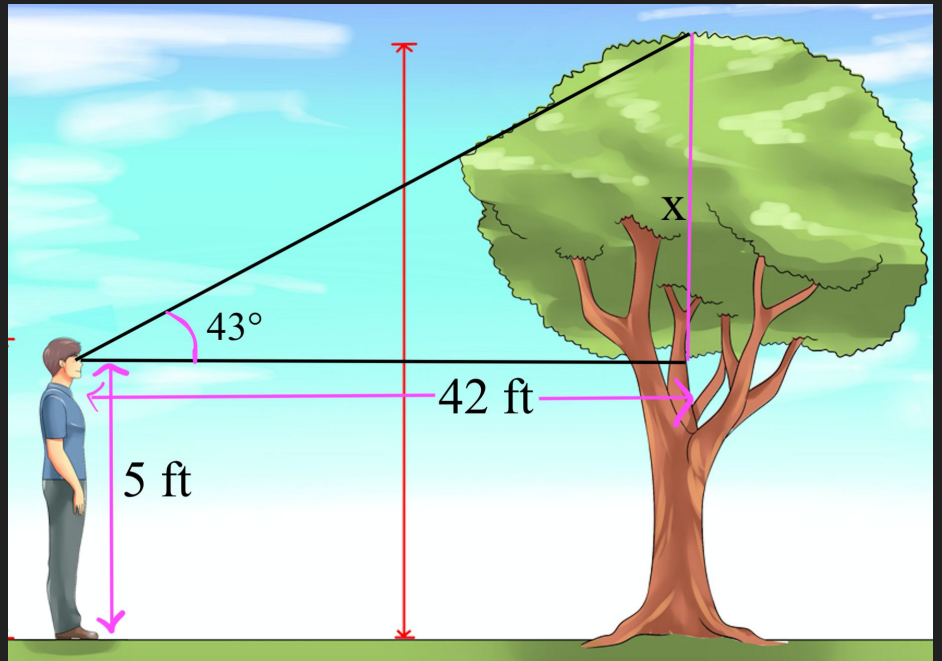

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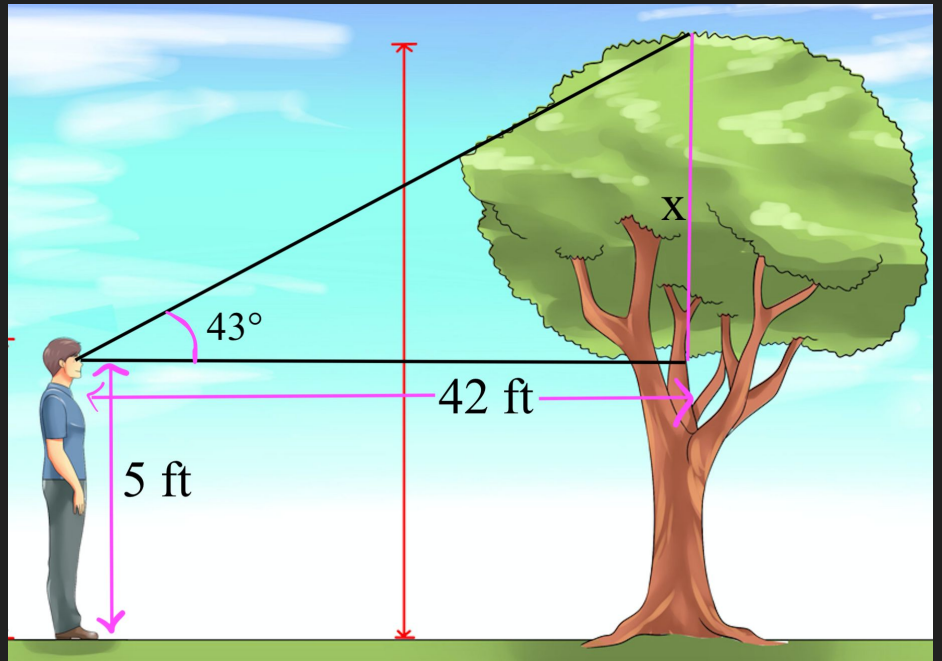

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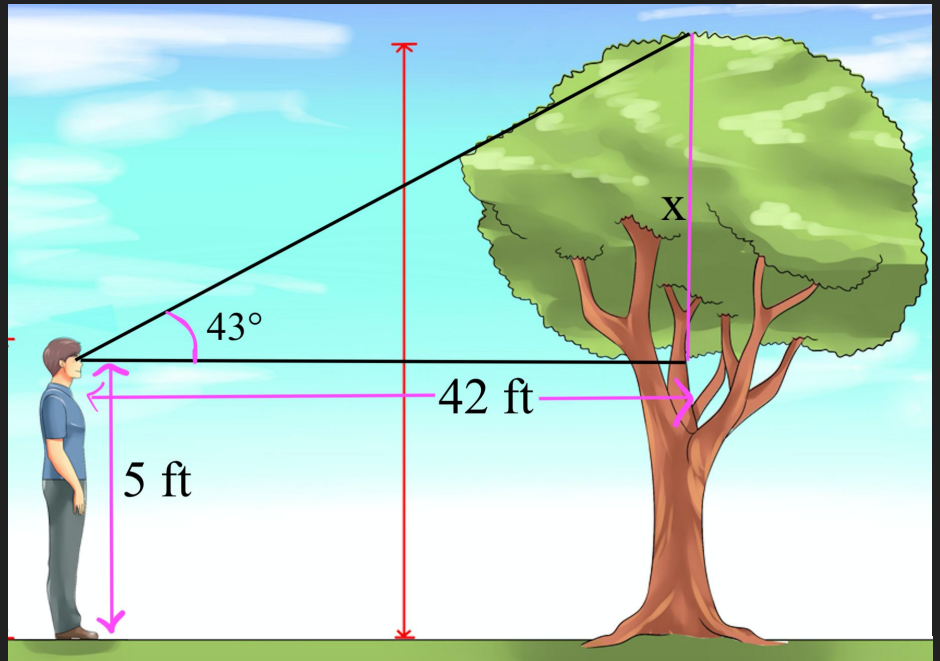

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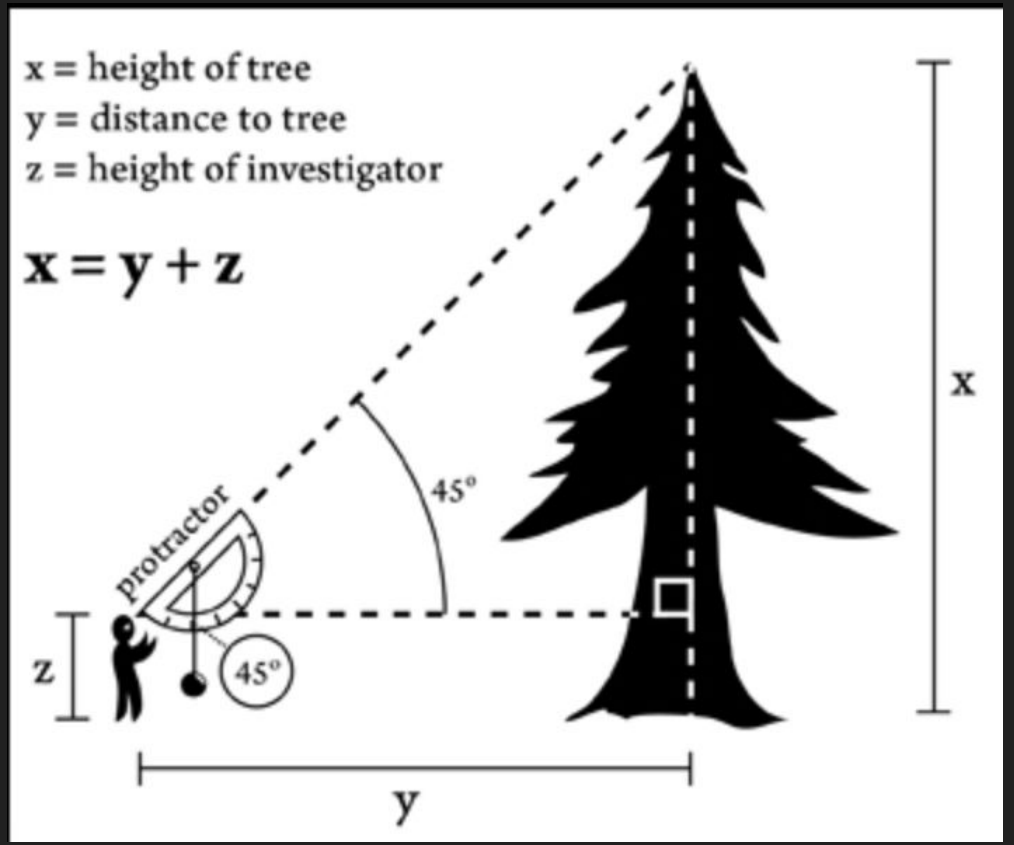
Tree Height = 39.2 + height from ground to line of sight

$$= 39.2 + 5 = 44.2\ \text{ft}$$

Now that we know how to solve using Trigonometric Ratios, we will find the height of this tree using both the Clinometer Method and the Stick Measurement Method.

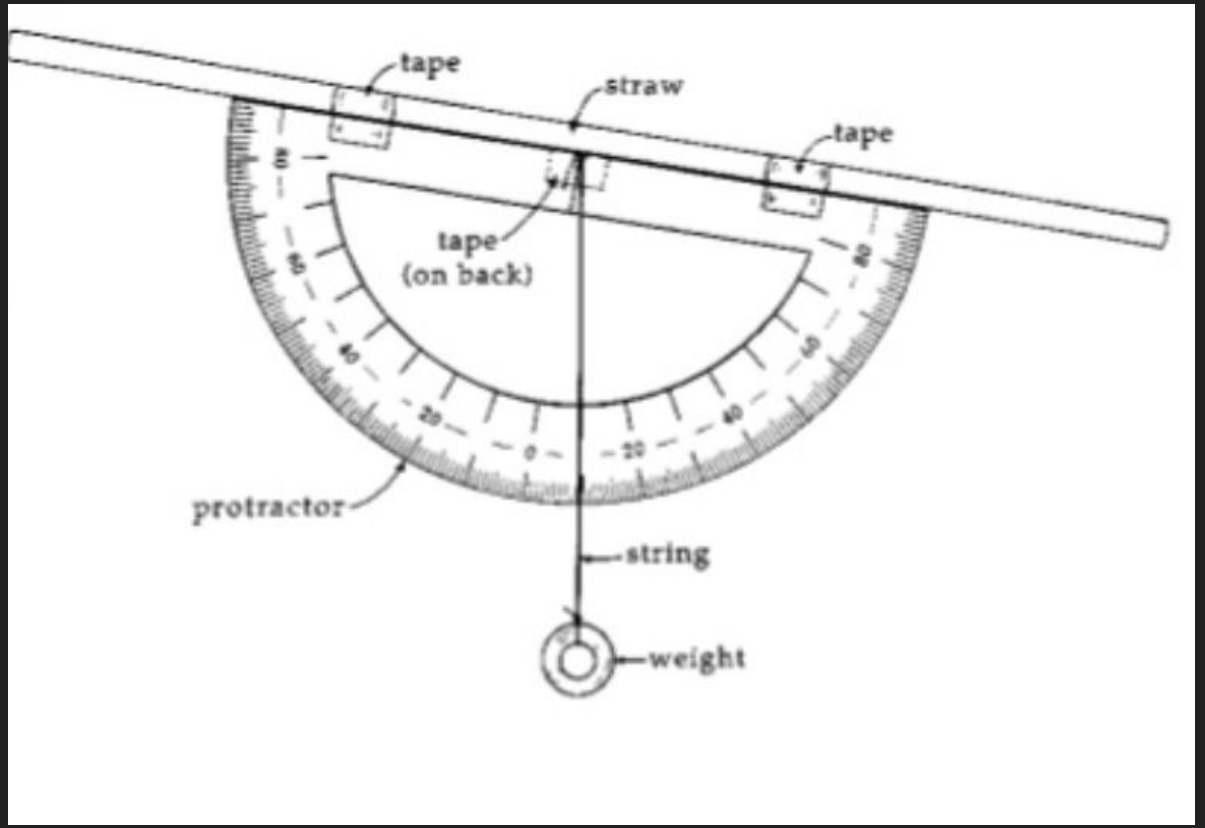
Clinometer Method:

In order to measure the angle from your eye-height to the top of the tree, also known as the **Angle of Elevation**, we must use a Clinometer!



You will need:

- Protractor
- Straw
- Tape
- String
- Weight



Look through the straw to the top of the tree and pinch your string where it falls. Record the acute angle measure.

Clinometer Method Example:

You are standing 24.5 ft away from the base of a tree. Your line of sight is 5.08 ft from the ground. If you find the measure the angle of elevation to the top of the tree using your clinometer, how tall is the tree?

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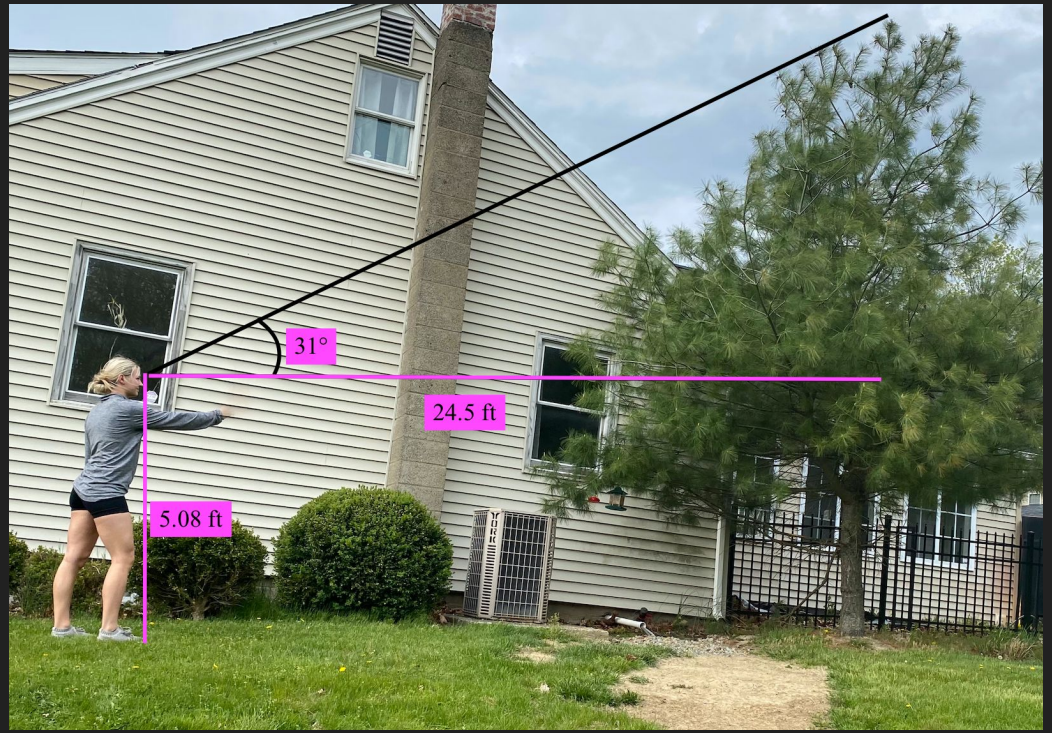

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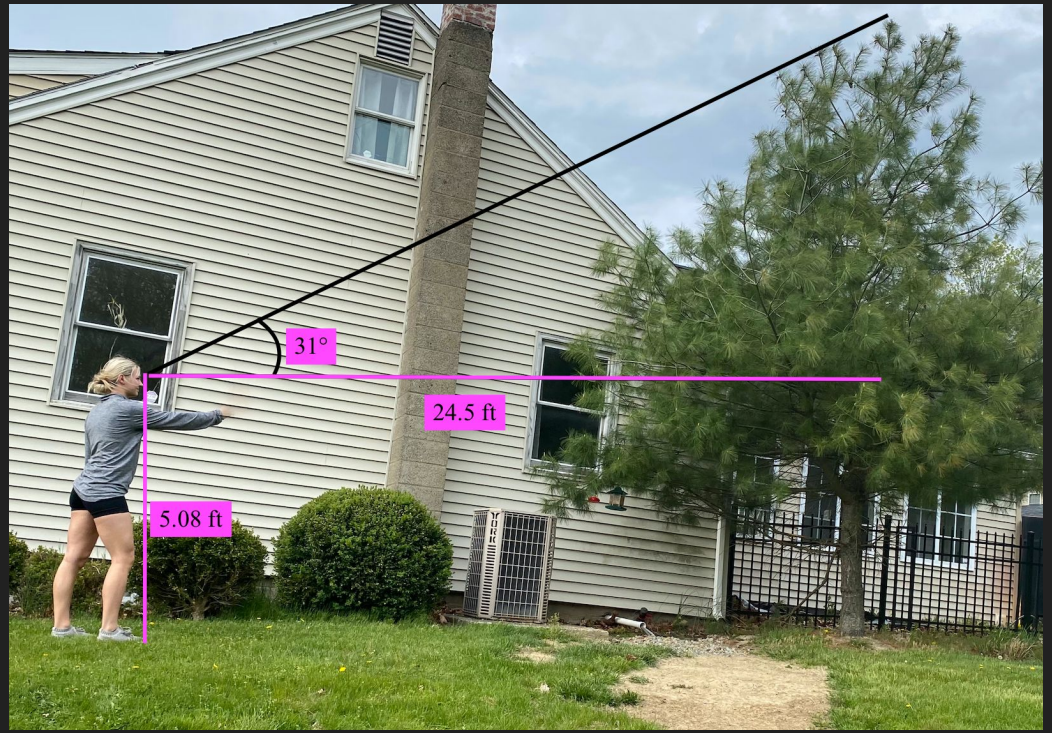

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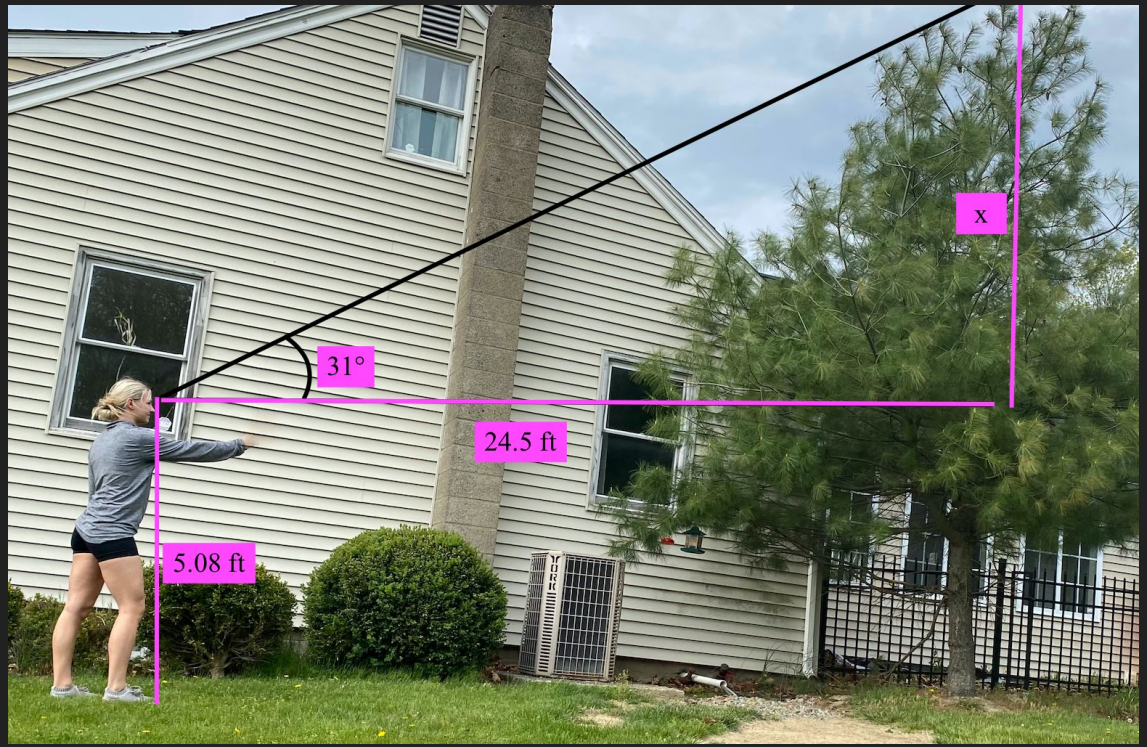

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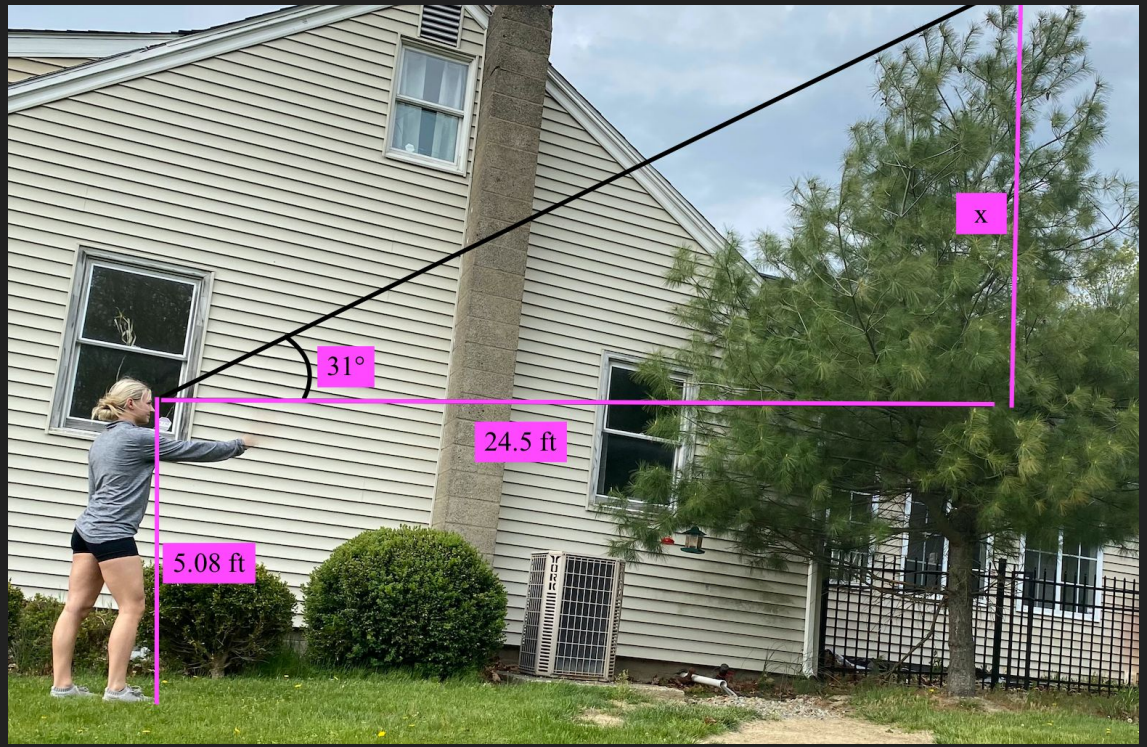

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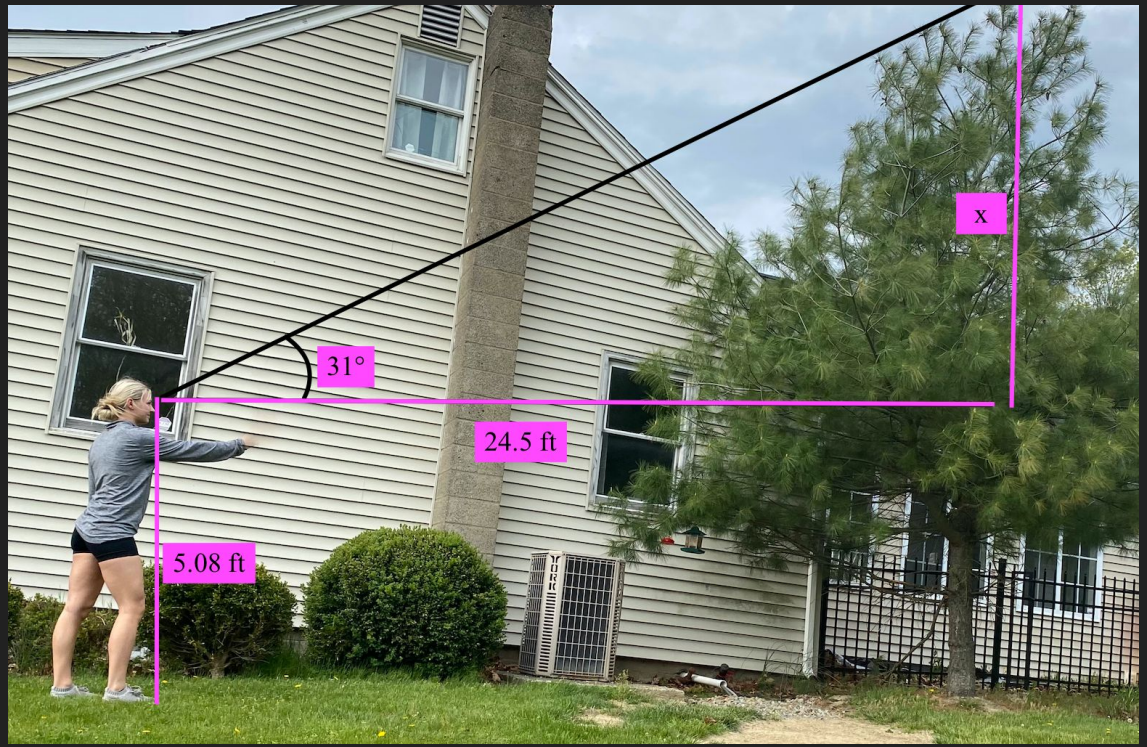

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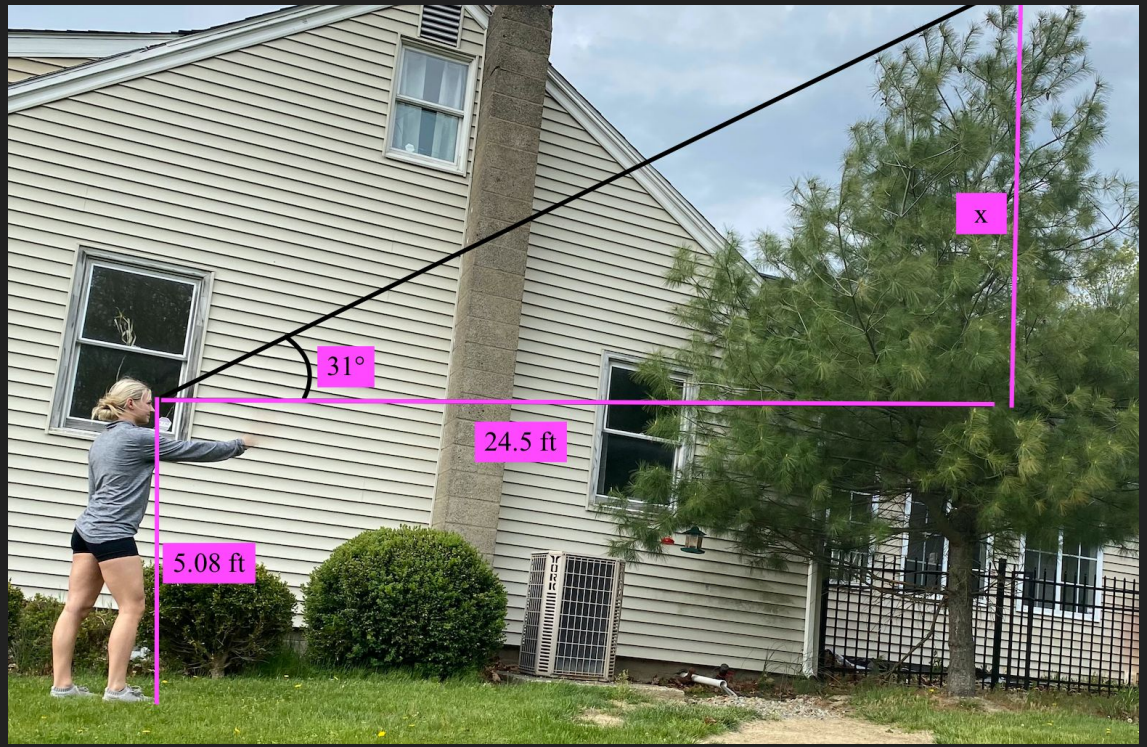

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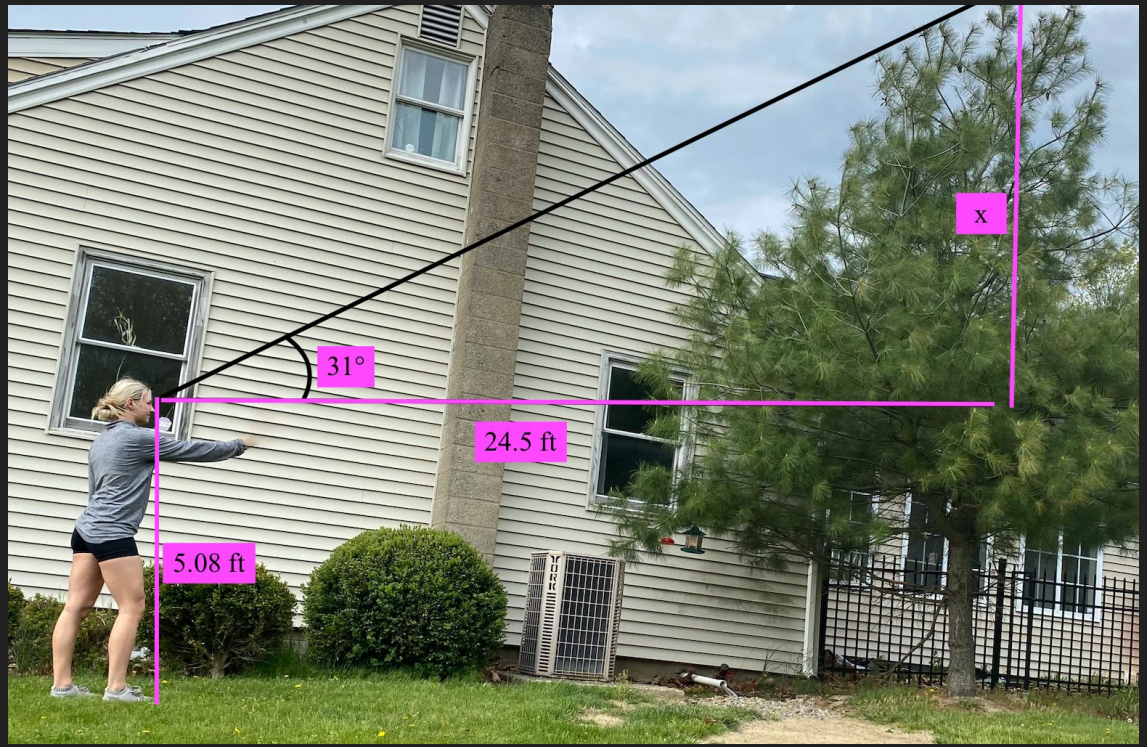

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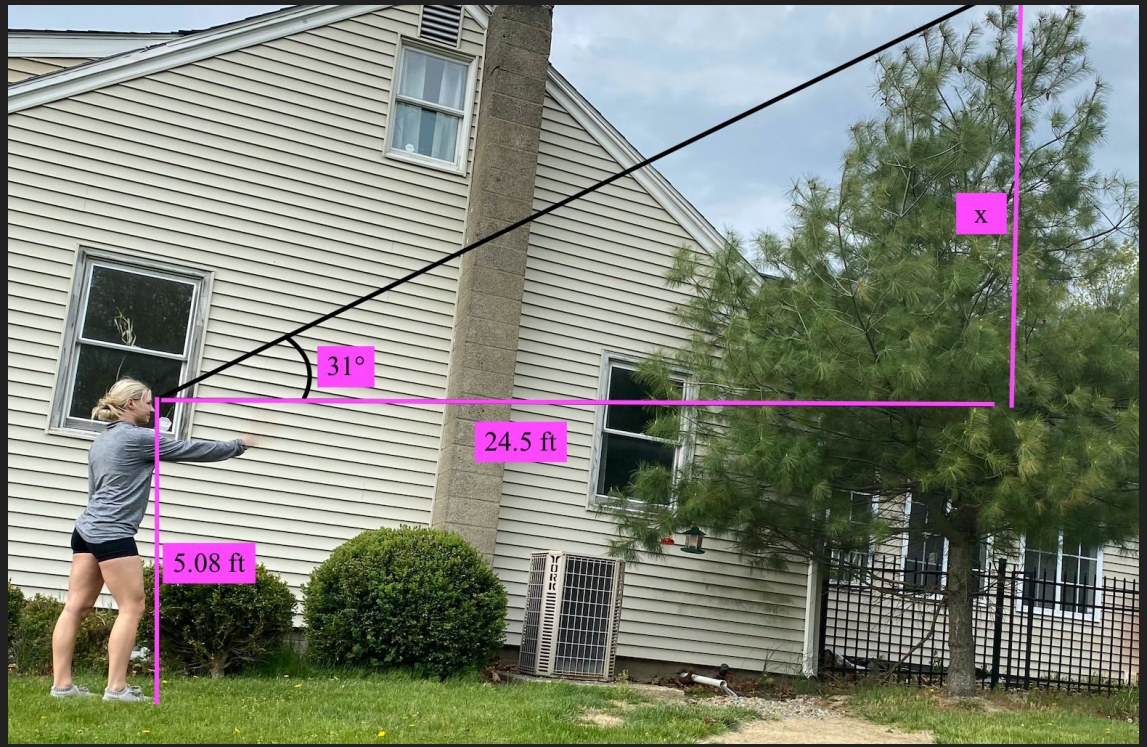

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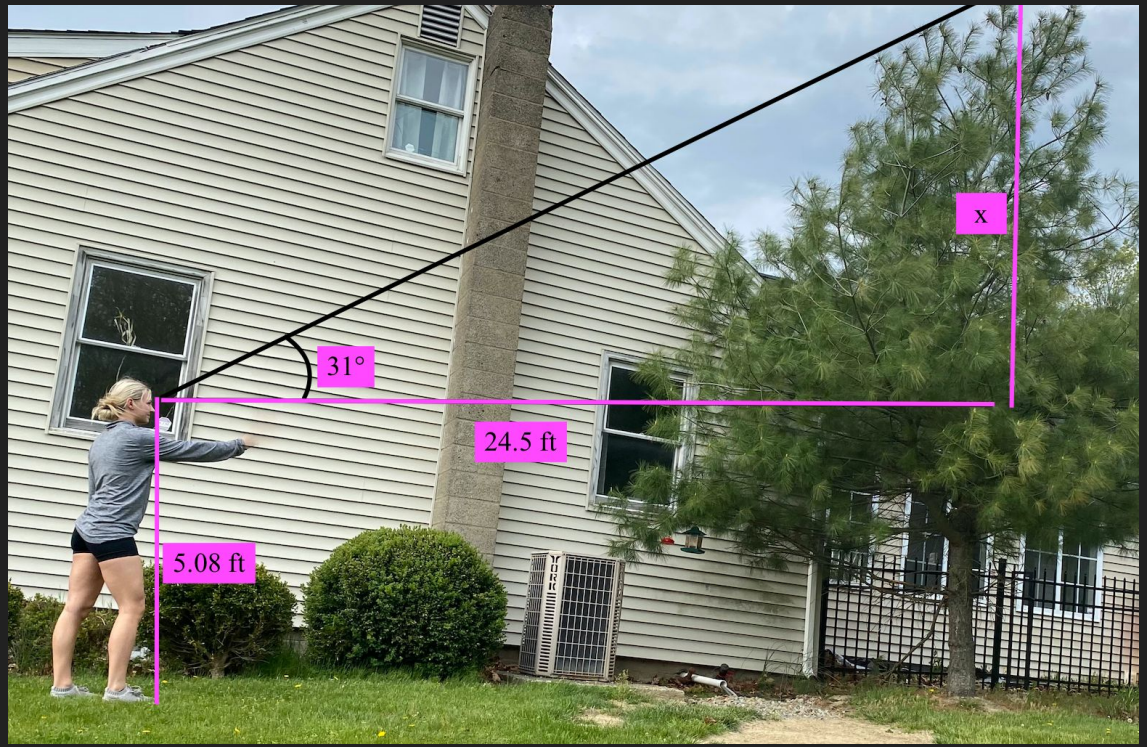

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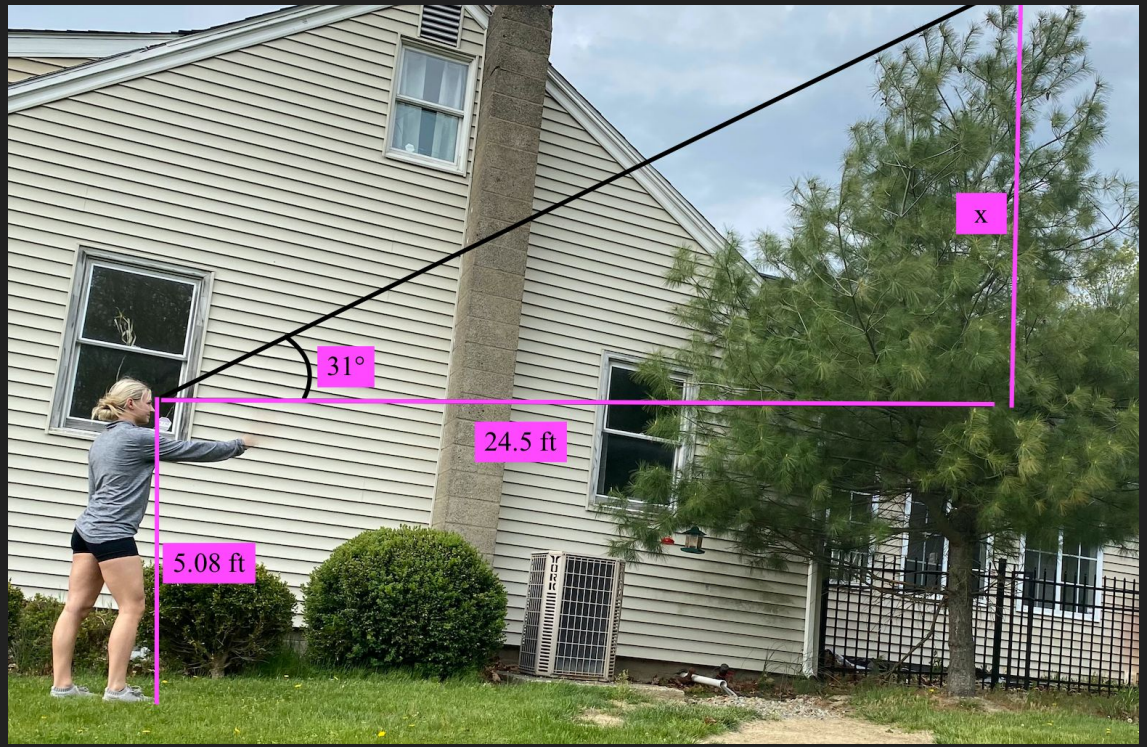

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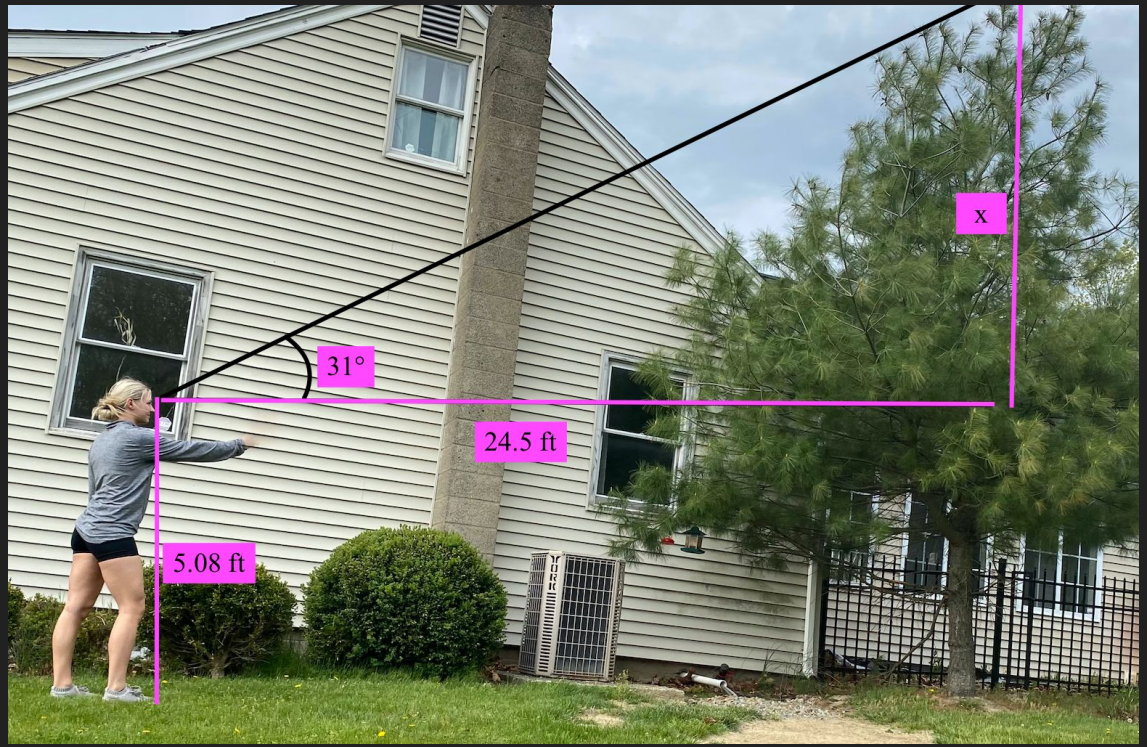

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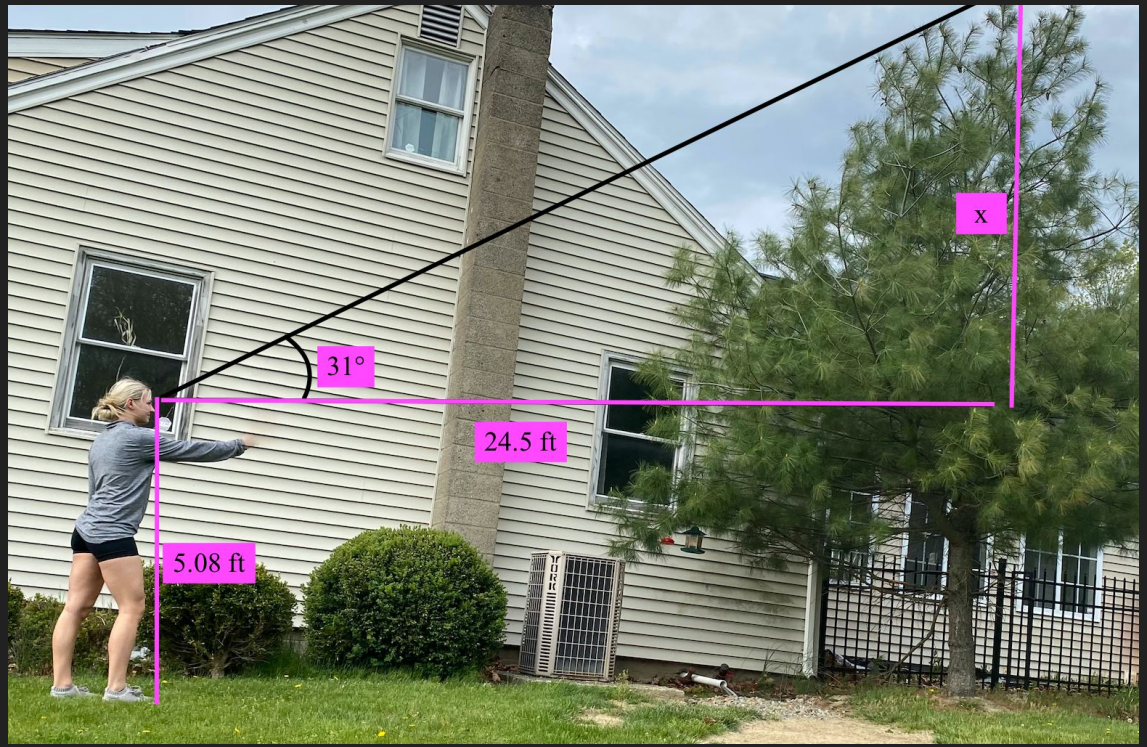

The total height of the tree is my eye height + x

5.08 ft + 14.7 ft

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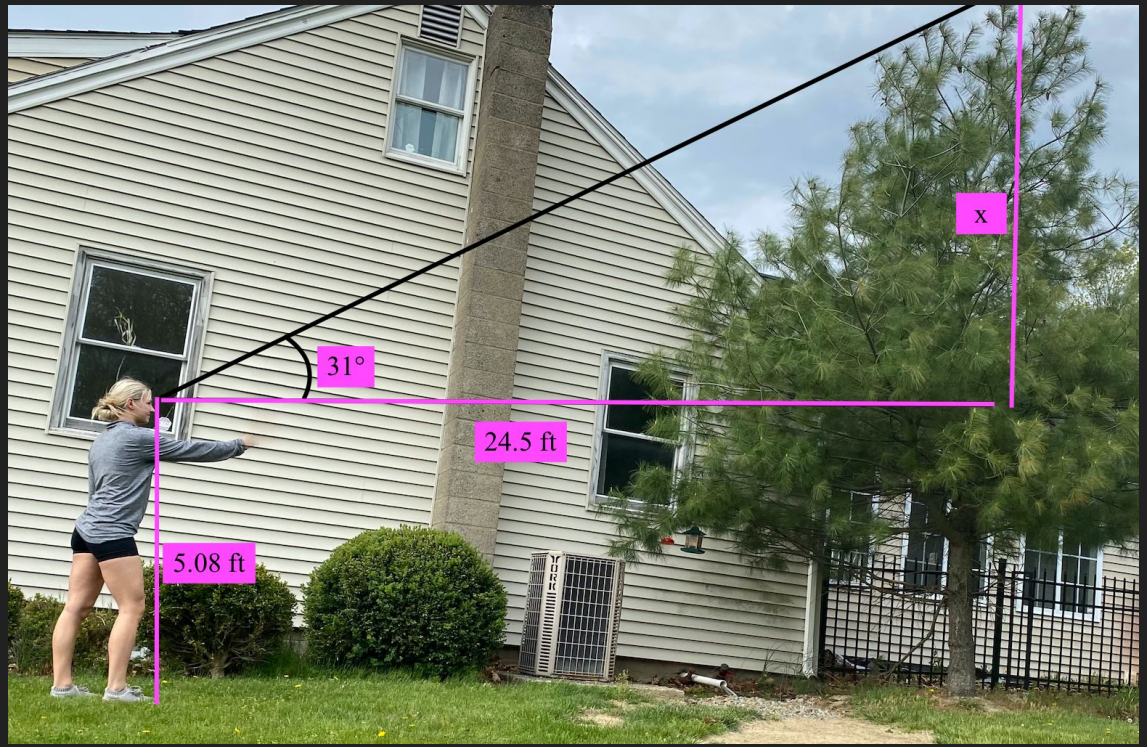

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$$5.08 \text{ ft} + 14.7 \text{ ft} = 19.8 \text{ ft}$$

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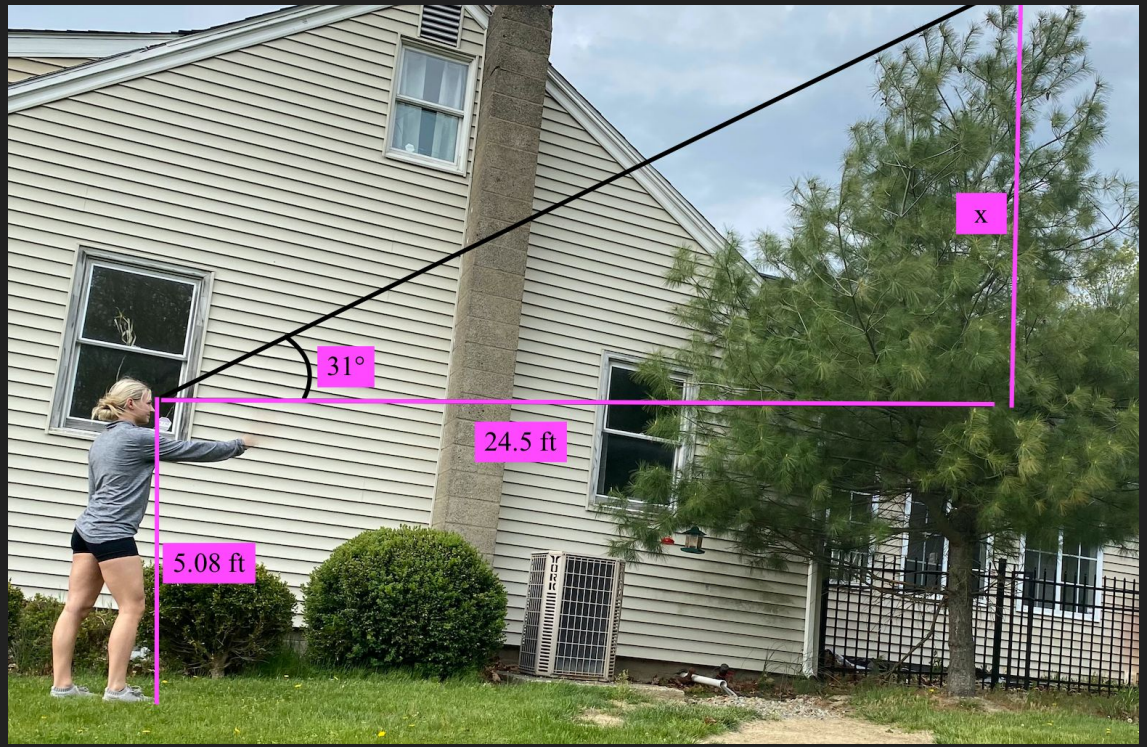


The total height of the tree is my eye height + x

$$5.08 \text{ ft} + 14.7 \text{ ft} = 19.8 \text{ ft}$$

Therefore, the tree is 19.8 ft tall.

Stick Measurement Method

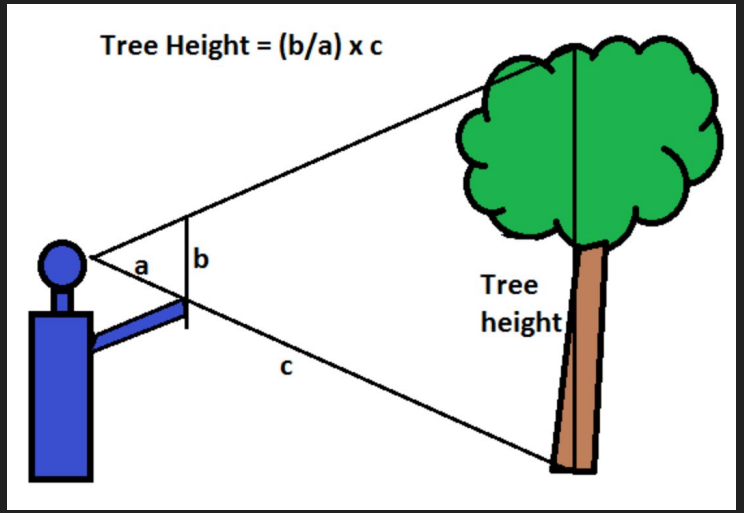
Hands-On Activity Using Tree!

1) Find a straight stick or ruler

2) Hold the stick vertically at arm's length, making sure that the length of the stick above your hand equals the distance from your hand to your eye.

3) Walk backward away from the tree. Stop when the stick above your hand exactly masks the tree.

4) Measure the straight-line distance from your eye to the base of the tree. Record that measurement as the tree's height to the closest foot. As with A, if the top is not vertically over the base, this method will generate an error.



Real Life Example of the Stick Measurement Method :

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Find a ‘straight’ stick or ruler

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Step 3.

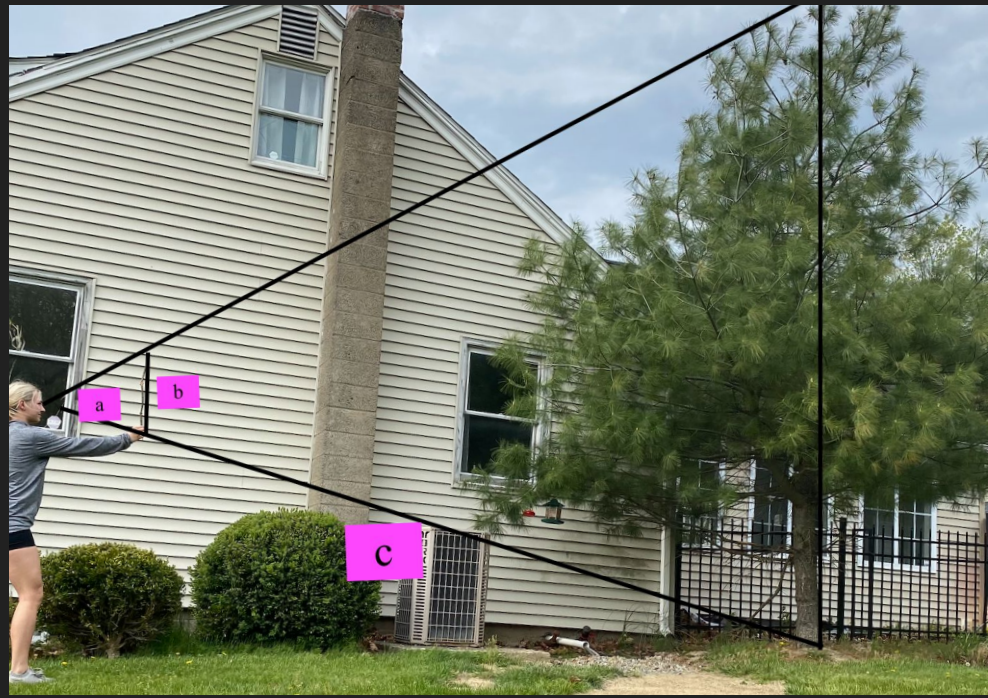
Walk backward away from the tree. Stop when the stick exactly masks the height of the tree.

Step 4.

Measure the distances of a, b, and c

a : the distance from your eyes to the base of your stick.

b : the length of your stick



c : the distance from your eye to the base of the tree.

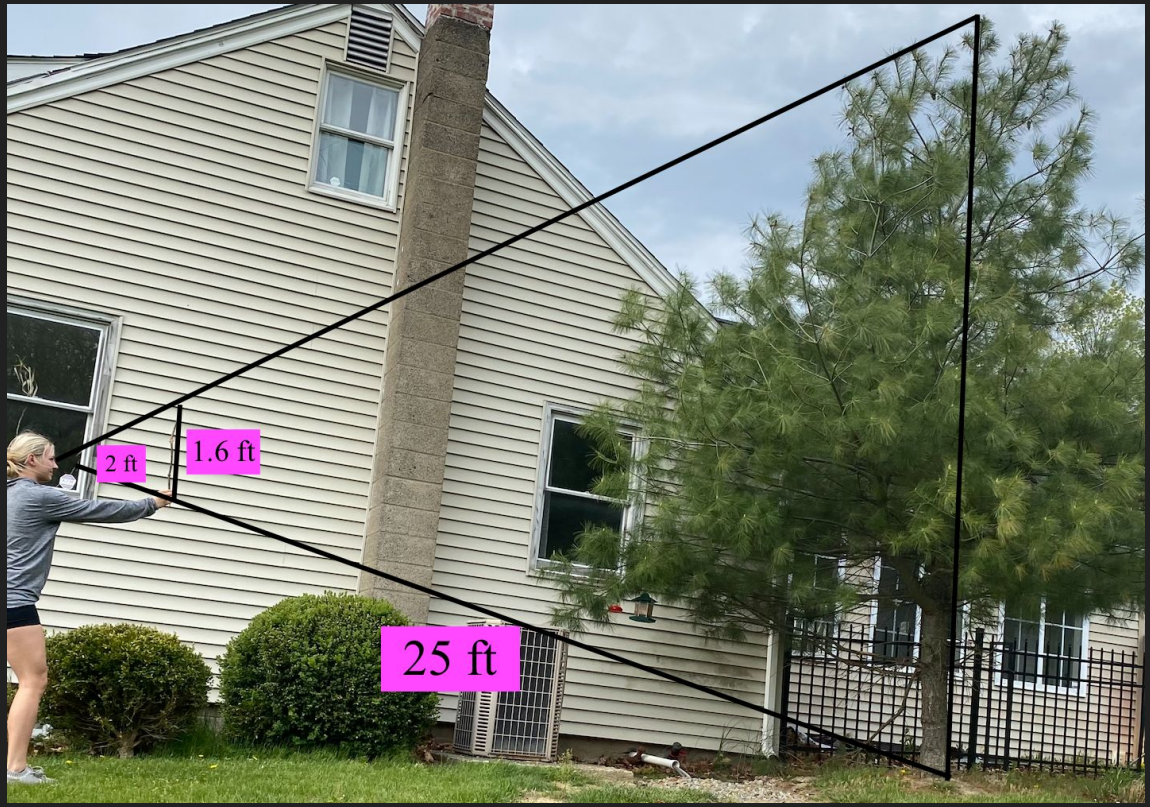
Step 4.

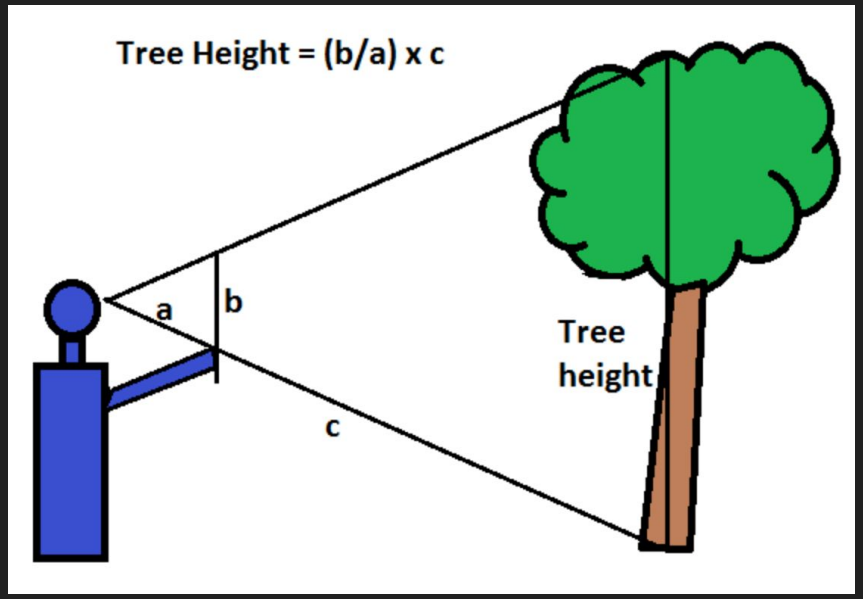
Measure the distances of a, b, and c

a = 2 feet

b : 1.6 feet

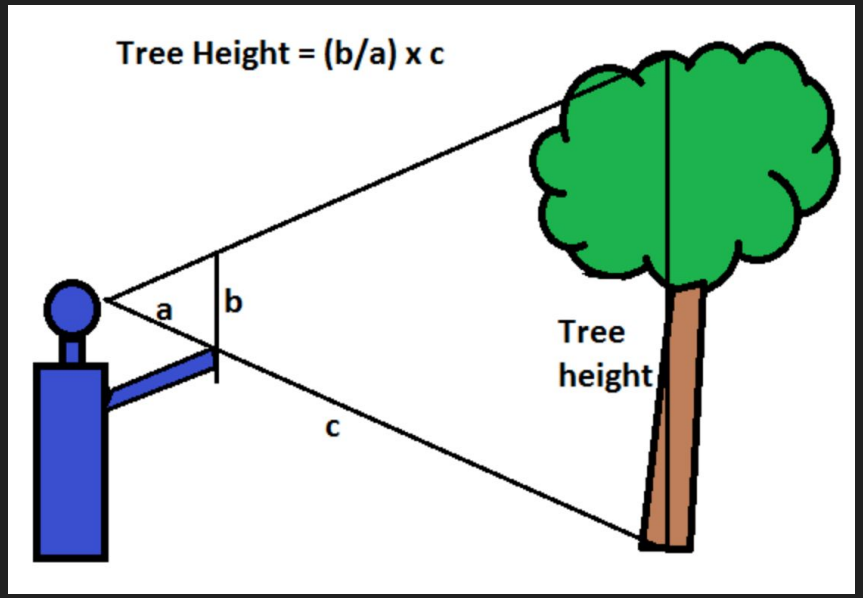
c : 25 feet





$$\text{Tree Height} = \frac{b}{a} \times c$$

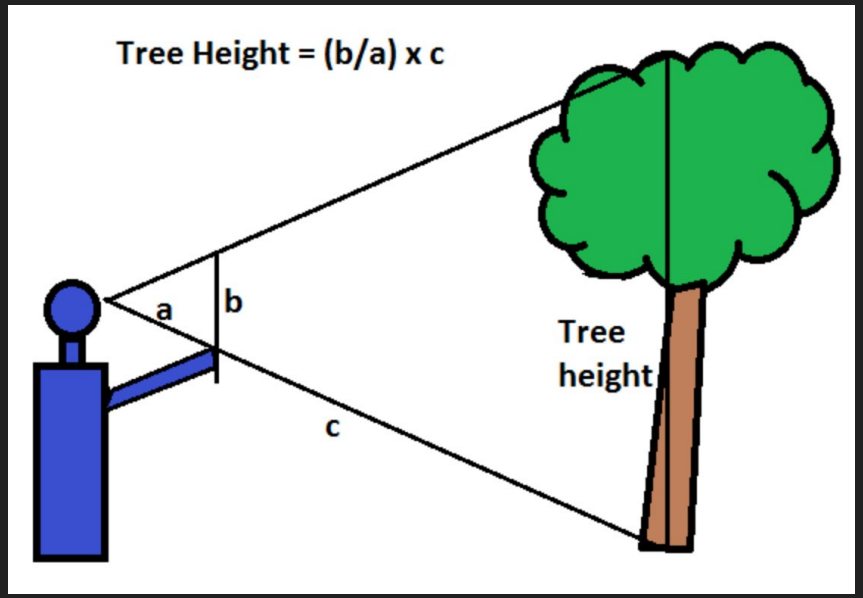
$$\text{My Tree Height} = \frac{1.6\text{ ft}}{2\text{ ft}} \times 25\text{ ft}$$



Tree Height = $\frac{b}{a} \times c$

My Tree Height = $\frac{1.6 \text{ ft}}{2 \text{ ft}} \times 25 \text{ ft}$

$= 0.8 \times 25 \text{ ft}$



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$$= 0.8 \times 25\text{ ft}$$

$$= 20\text{ ft tall}$$

Finally, let's calculate our percent difference from our answers using the Trigonometry versus the Stick Method.

$$C = \frac{x_2 - x_1}{x_1} \times 100$$

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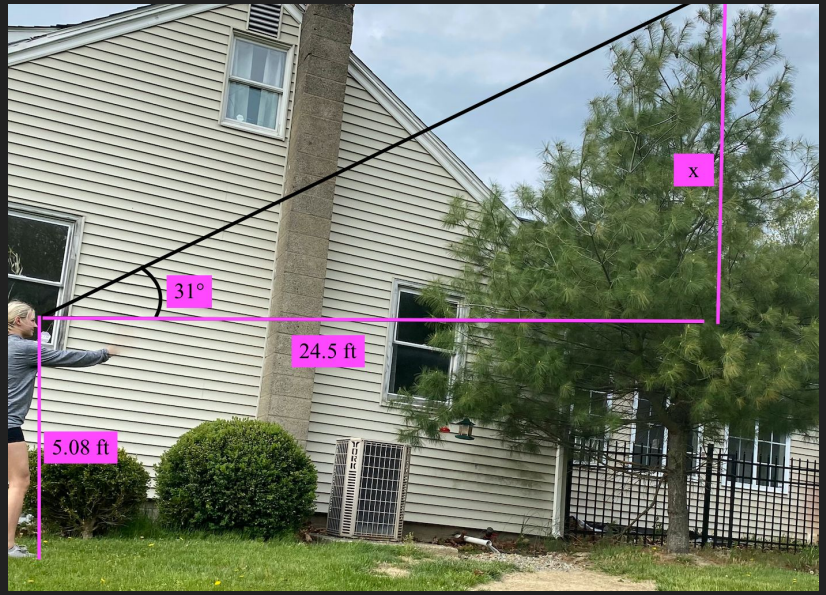
Finally, let's calculate our percent difference from our answers using the Trigonometry versus the Stick Method.

$$C = \frac{x_2 - x_1}{x_1} \times 100$$

$$C = \frac{20 - 19.8}{19.8} \times 100 = 1.01\ \%$$



Method using Clinometer

Method using Stick Measurement

Now that we know two methods on how to find the height of a tree, it's your turn to apply this to your own tree!

“Tree Project”

After presenting this lesson on how to find the height of a tree using the Clinometer Method and the Stick Method, I would give the students an assignment using both methods to check their understanding of the lesson.

Tree Project

1. Students will be split into pairs

2. Each group of two must find a tree that has a base on a flat ground where these measurements can be calculated, similar to the tree I used.

3. Each group must find the height of their tree, using both the Stick Method and the Clinometer Method, and find their percent difference.

4. You only need to show the work for measurements of one member of your group. So one of you will stand and the other will measure.

- If I were to utilize this lesson in a classroom setting, I would use a tall object on the wall, which can be a tree on the wall, like the photo, or even using the corner of a room.

- Each student has a different eye height, therefore all of their variables will be different, but should all end up with the same answer for the height of the tree.