Name: _____

Desmos Lesson: Graphing & Interpreting Functions: Pre- Calc / Calculus I

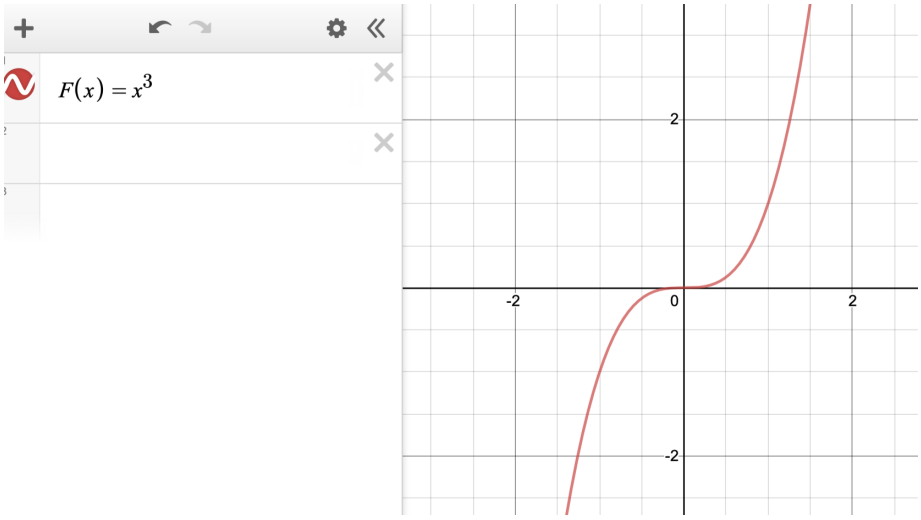
Graphing Polynomials: End Behavior · Identifying Turning Points · Multiplicity

1.) Open https://www.desmos.com in your browser
2.) Click the "Graphing Calculator" button.

I. End Behavior

The **end behavior** of a function *f* describes the behavior of the graph of the function at the "ends" of the *x*-axis. Follow along below to see how to write behaviors in the proper notation.

In your cursor on Desmos, type in the function : $f(x) = x^3$



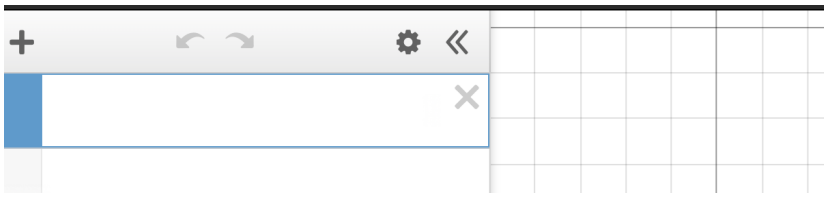
To describe the end behavior of this function, we write it in the notation:

As $x \to -\infty$, $f(x) \to -\infty$: This means: As *x* approaches negative infinity, *f(x)* approaches negative infinity.

As $x \to +\infty$, $f(x) \to +\infty$: This means: As *x* approaches positive infinity, *f(x)* approaches positive infinity.

1.) Clear your graph on Desmos. Type in each of the following functions, sketch the graphs, and describe their end behaviors.

Type your functions here →



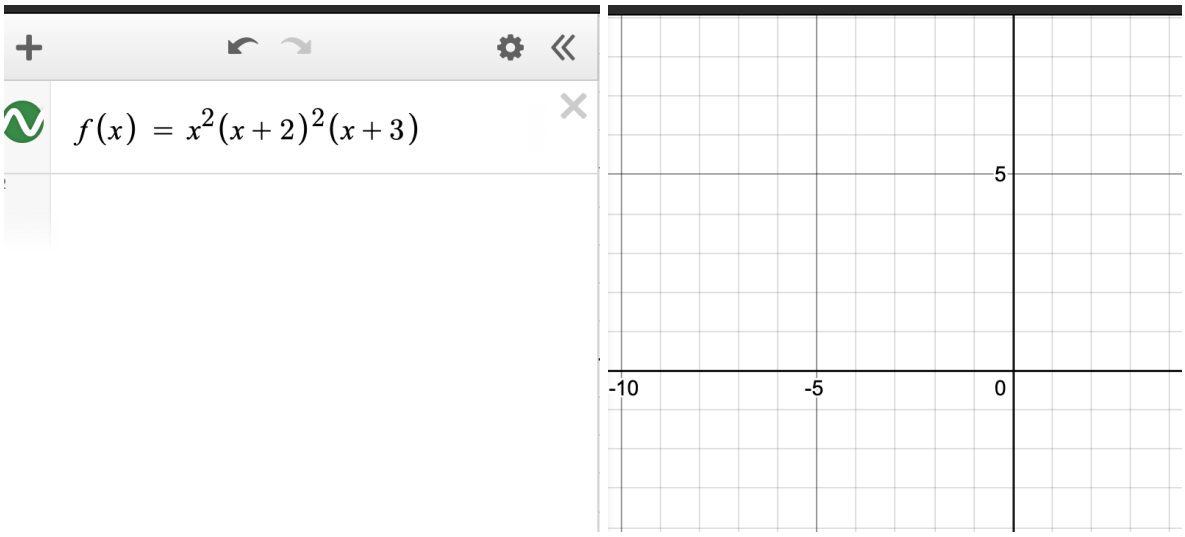
a) Graph in Desmos: $f(x) = x^2$ b) Sketch the function. c) Describe the end behaviors:	
a) Graph in Desmos: $f(x) = -x^7$ b) Sketch the function. c) Describe their end behaviors:	
a) Graph in Desmos: $f(x) = 3x+5$ b) Sketch the function. c) Describe their end behaviors:	

II. Identifying Turning Points (Extrema)

A **turning point** is a point on the graph where the graph changes from increasing to decreasing **or** decreasing to increasing. A polynomial of degree n will have at most **n – 1** turning points.

- **A function is increasing if** the function values increase as input values increase (as x increases, y increases).
- **A function is decreasing if** the function values decrease as the input values increase (as x increases, y decreases).

1.) In your cursor on Desmos, type in the function : $f(x) = x^2 (x + 2)^2 (x + 3)$



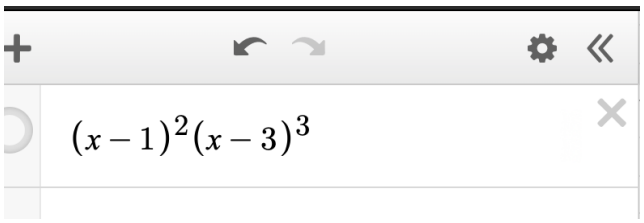
a) Sketch your graph on the axis above.
b) On what interval(s) is the function increasing? Decreasing? (round to the nearest tenth)

c) In a different color pencil, mark each turning point on your graph.
d) There are _____ turning points on this graph.
e) Recall the conjecture, "A polynomial of degree n will have at most **n – 1** turning points". Based on the equation, what number does n represent? _____

Does your answer to part "c" relate to this conjecture? Explain briefly using arithmetics.

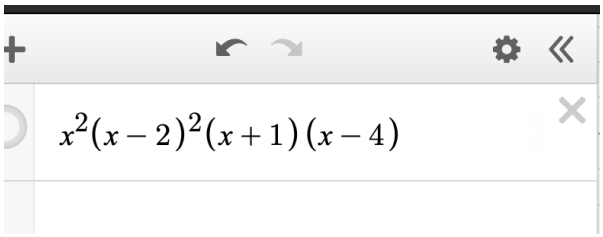
2.) Using Desmos, **sketch** and **identify** how many turning points are in the following examples.

a) **$f(x) = (x - 1)^2 (x - 3)^3$** **Sketch**



Amount of turning points: _____

b) **$x^2 (x - 2)^2 (x + 1) (x - 4)$** **Sketch**



Amount of turning points: _____

III. Zeros & Multiplicity

Zeros are where a function equals the value zero (0).

The **multiplicity** is the number of times a given factor appears in the factored form of the equation of a polynomial.

Key Graph Behaviors with Zeros & Multiplicities

- For zeros with **even multiplicities**, the graphs touch or are tangent to the x-axis.
- For zeros with **odd multiplicities**, the graphs cross or intersect the x-axis.
- For zeros with **multiplicity of zero**, the behavior near the intercept is like that of a line.

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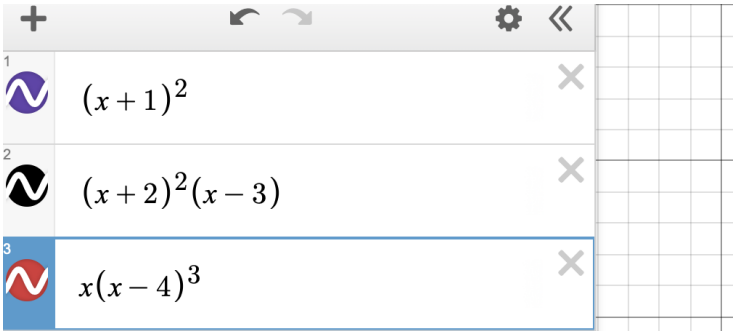
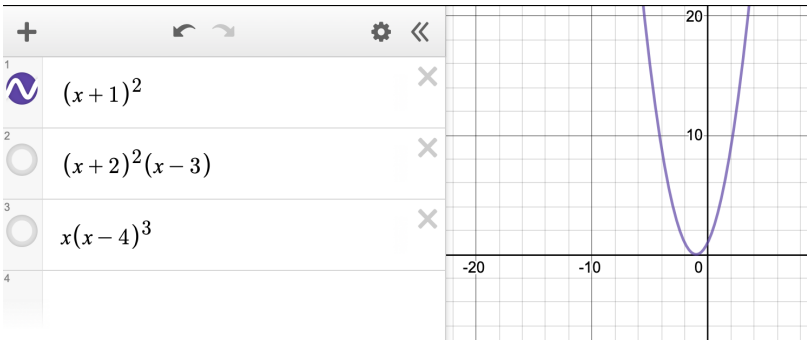
Let's Practice

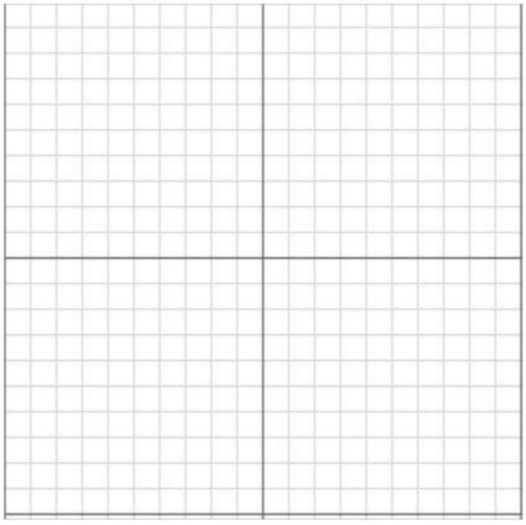
Clear all previous work in your Desmos calculator. Along the left side, type in the three equations: $f(x) = (x + 1)^2$, $f(x) = (x +2)^2(x - 3)$, $f(x) = x(x - 4)^3$



Currently, you should have three graphs overlapping each other. In order to only view one graph at a time, click on the colored circles to the left of the equations. Only the one with color will appear on the graph. If viewing only the first equation, your screen should look like this:



1.) Sketch & Interpret the first graph, $f(x) = (x + 1)^2$

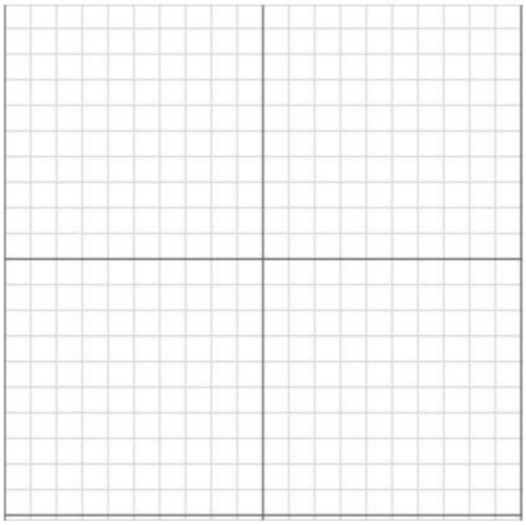


Solve for the zero(s) of the function:

What are the multiplicities of the zero(s)?

Describe the behavior of the zero(s) on the graph and why this occurs.

2.) Sketch & Interpret the second graph, $f(x) = (x +2)^2 (x - 3)$

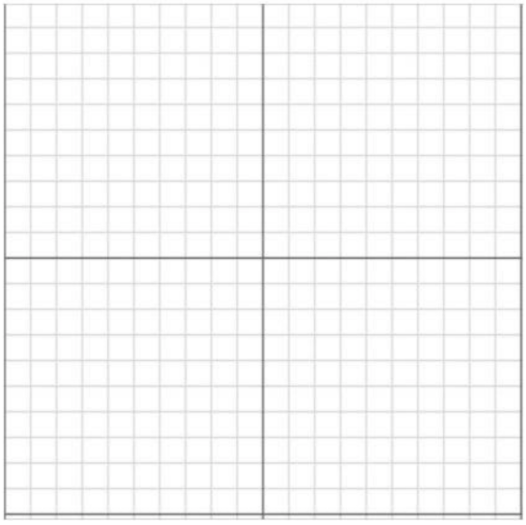


Solve for the zero(s) of the function:

What are the multiplicities of the zero(s)?

Describe the behavior of the zero(s) on the graph and why this occurs.

3.) Sketch & Interpret the first graph, $f(x) = x(x - 4)^3$



Solve for the zero(s) of the function:

What are the multiplicities of the zero(s)?

Describe the behavior of the zero(s) on the graph and why this occurs.

Final Activity: Bringing it all Together :

Enter the function in the Desmos cursor: **$f(x) = (x + 3)(x - 2)^2(x + 1)^3$**

Describe (in detail) this graph's:

a) End Behavior Along the x-axis (Part I)

b) How many turning points are there? (Part II)

c) Zeros, their multiplicities, and their behaviors (Part III)

Citations

https://www.desmos.com/calculator

https://courses.lumenlearning.com/waymakercollegealgebra/chapter/multiplicity-and-turning-points/